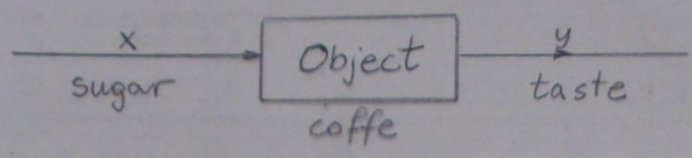


Synthesis of an one-argument fuzzy model describing the dependence $y = f(x)$
 $\text{coffe taste} = f(\text{sugar amount})$



Numerical domains of variables

sugar $S \in [0, 10]$ coffe-spoons
 taste $T \in [0, 10]$ points

Linguistic domains (vocabularies) of variables

sugar $S \in \{\text{small, average, great, very great}\}$ amount
 taste $T \in \{\text{poor, mean, good, very good}\}$

Individual definitions (understanding) of linguistic values by an investigated person (AP)

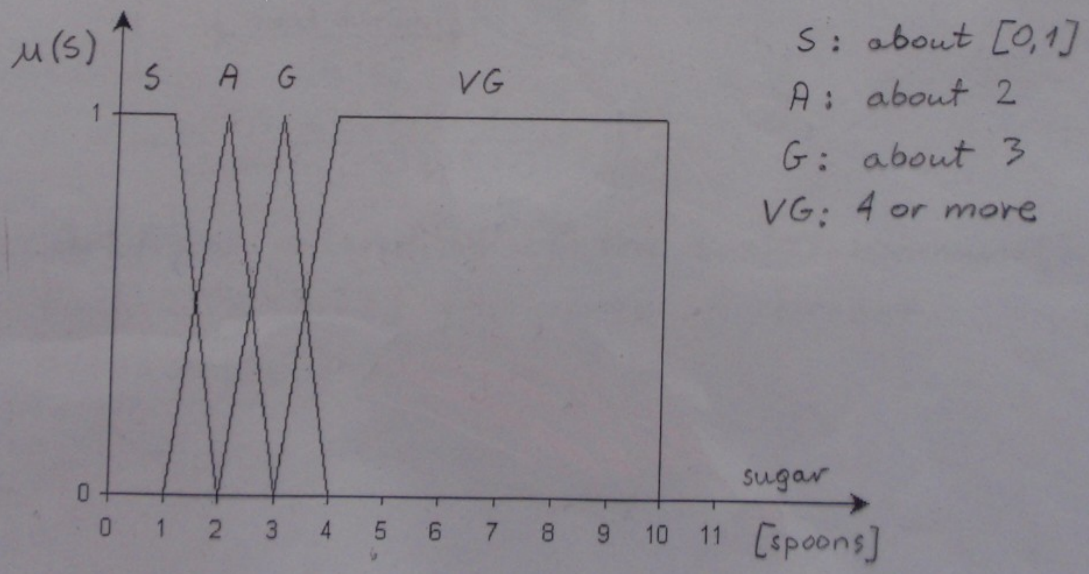


Fig. 1. Membership functions of linguistic values describing amount of sugar put in a coffee

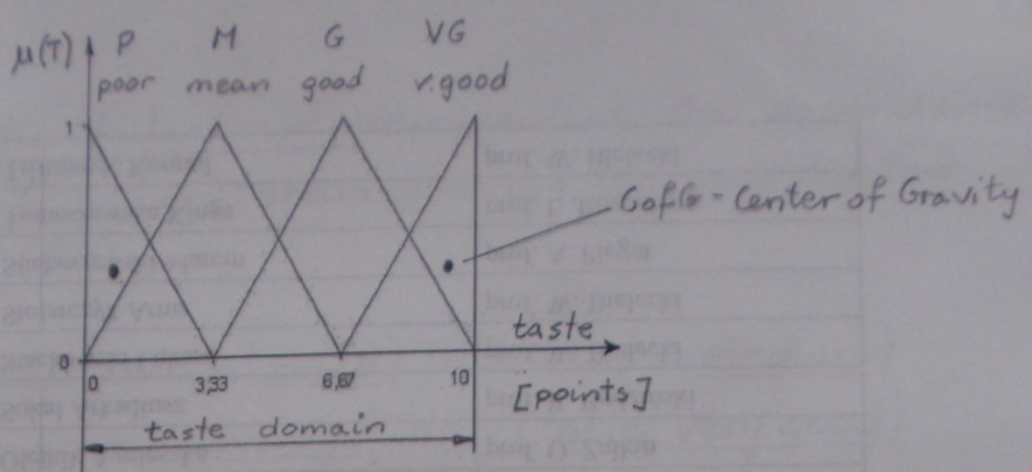


Fig. 2. Membership functions of linguistic values describing taste of coffe

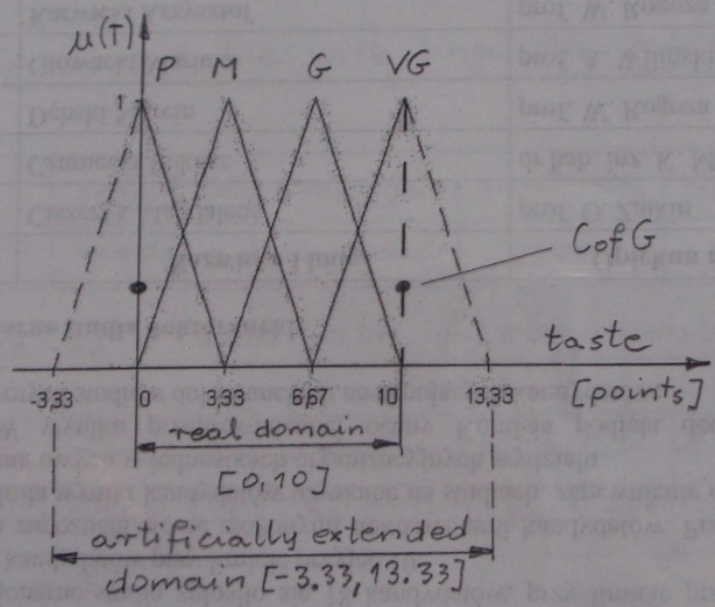


Fig. 3. Artificial extending of the taste-domain [0,10] to [-3.33, 13.33] for aims of correct defuzzification.

Linguistic Rule-Base representing the dependence $Taste = f(Sugar)$ characterizing the investigated person (AP).

R1): IF (sugar amount small) THEN (taste medium)

R2): IF (sugar a. average) THEN (taste very good)

R3): IF (sugar a. great) THEN (taste good)

R4): IF (sugar a. v.great) THEN (taste poor)

Rule-Base as a table with linguistic values

sugar	small am.	average a.	great am.	v.great am.
taste	medium	very good	good	poor

Rule-Base as a table with fuzzy numbers

sugar	about [0,1]	about 2	about 3	4 or more
taste	about 3,33	about 10	about 6,67	about 0

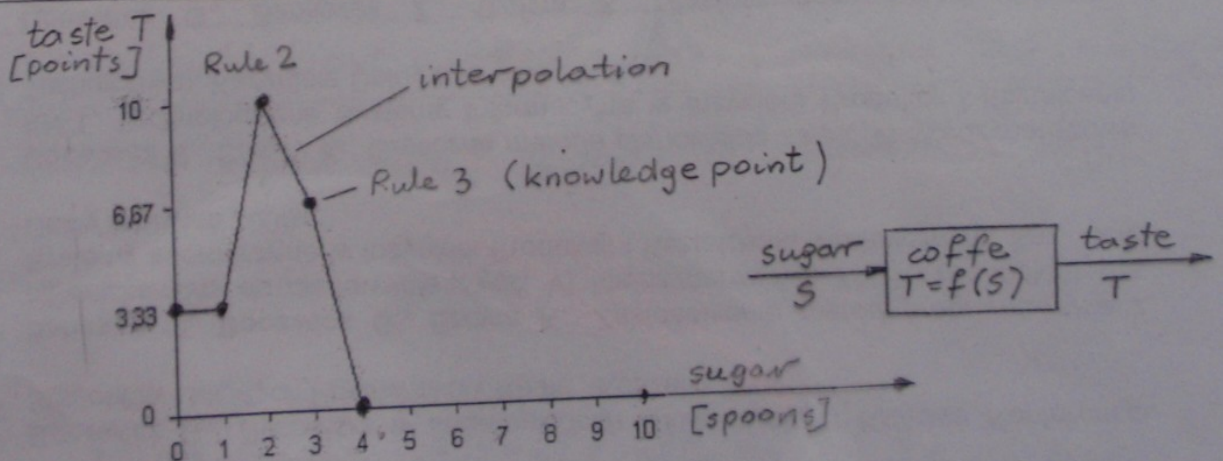


Fig. 4. Visualization of the fuzzy model of the dependence $T=f(s)$ existing in the investigated person (1-person model).

(4)

What a complete fuzzy model should contain to enable calculations and its practical using?

1. All membership functions defining linguistic values used in the model (as small amount of sugar, medium taste, etc)
2. Rule-base linguistically characterizing the modeled dependence $y=f(x)$ (taste as function of sugar amount,
3. Inference method in a single rule (e.g. implication operator of Mamdani - Minimum) that delivers a single-rule conclusion
4. Aggregation method of all single-rule conclusions in one general conclusion (e.g. application of the Maximum-operator)
5. Defuzzification method of the general conclusion for achieving a one-value representation of this conclusion (e.g. defuzzification with the Center of Gravity- or the Singleton-method).

Example calculation with the fuzzy model taste = f(sugar

I am intending to put 2.25 spoons of sugar in my coffee cup.

What will be the coffee taste?

There is no rule of the form:

IF (sugar $C=2.25$ spoons) THEN (coffee taste $S=...$ points)

Inference according to Rule 1

R1: IF (sugar amount small) THEN (coffee taste medium)

What is the opinion of Rule 1 concerning 2,25 spoons of sugar?

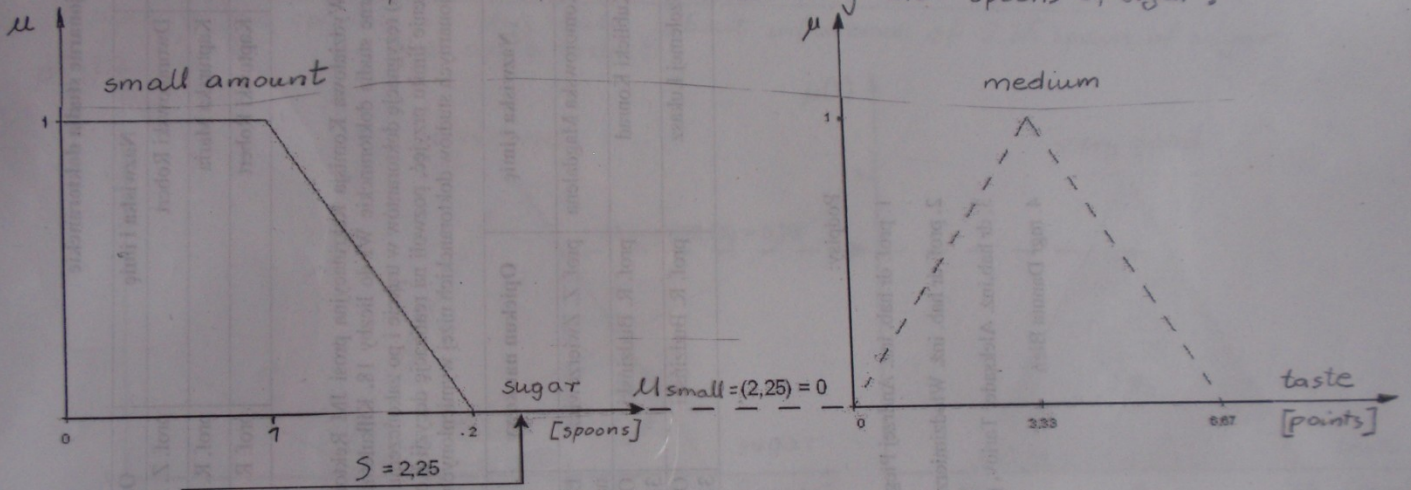


Fig. 5

Comment:

2.25 spoons of sugar has property of the "small" amount of sugar in degree 0 (it doesn't possess this property). Therefore the conclusion "medium" taste wasn't activate at all. Rule 1 doesn't concern the sugar-amount 2.25 spoons.

Inference according to Rule 2

⑥

R2: IF (sugar amount average) THEN (taste very good)

What is the opinion of this rule about influence of 2,25 spoons of sugar on the coffee taste?

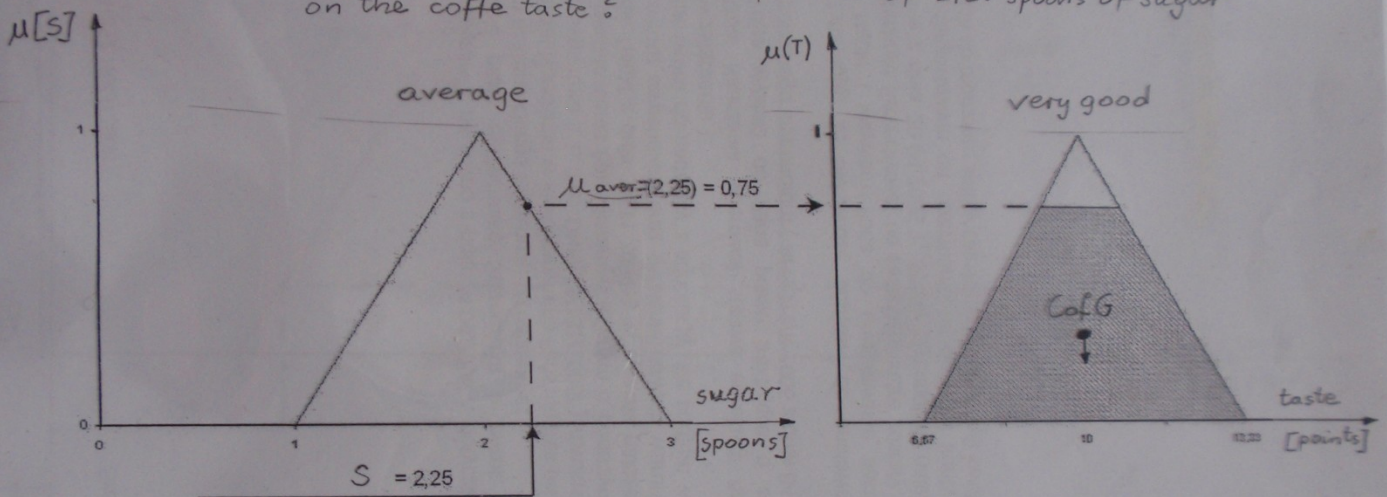


Fig. 6

Comment:

2.25 sugar-spoons possess the property "average" to degree 0.75. Therefore the conclusion taste "very good" was activated also to degree 0.75. Rule 2 predict with strength 0.75 that coffee taste will be very good.

Inference according to Rule 3

Rule 3: IF (sugar amount great) THEN (coffee taste good)

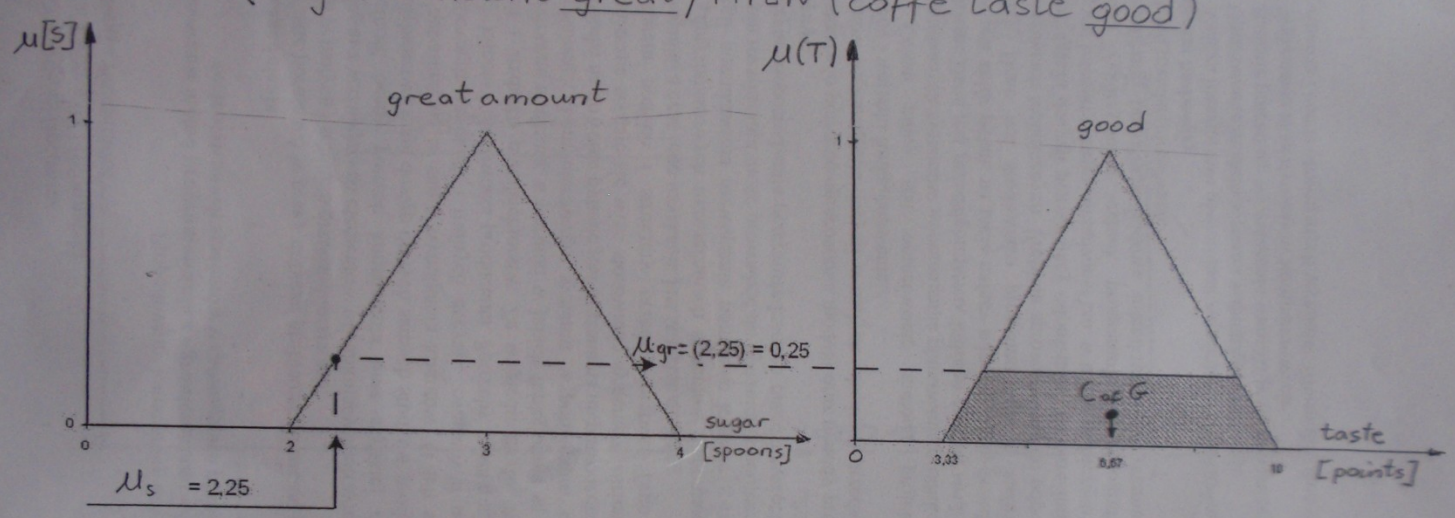


Fig. 7

Comment:

2.25 spoons of sugar means "great" amount to degree 0.25. Thus, prediction that the taste will be "good" has strength of 0.25.

Inference according to Rule 4

Rule 4: IF (Sugar amount very great) THEN (coffee taste poor)

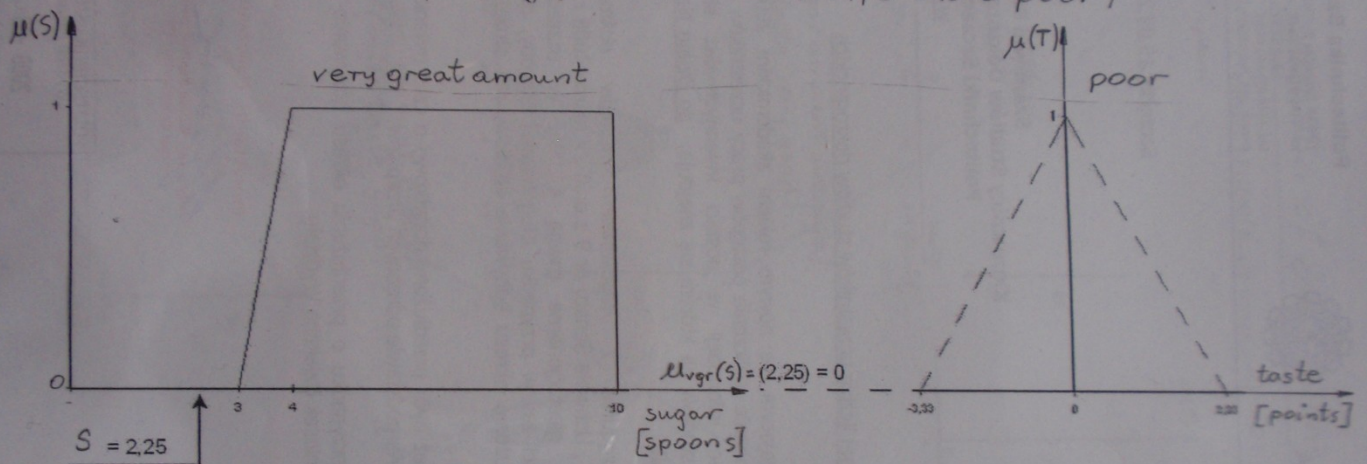


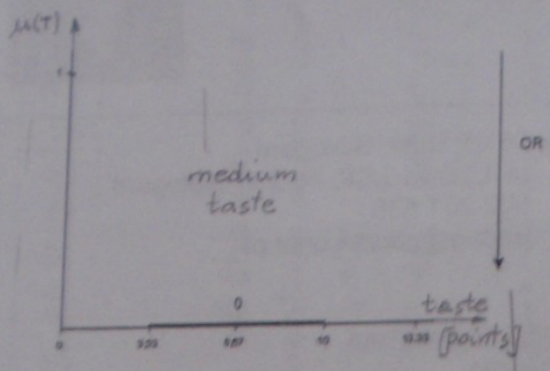
Fig. 8.

Comment:

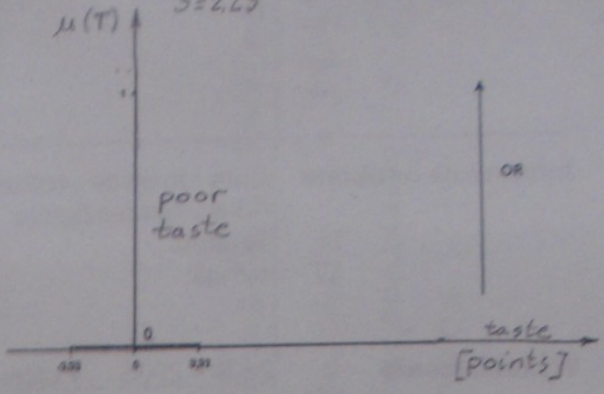
2.25 sugar-spoons don't possess property "very great" amount. Therefore conclusion "poor" taste was not activated.

Aggregation of conclusions of all rules

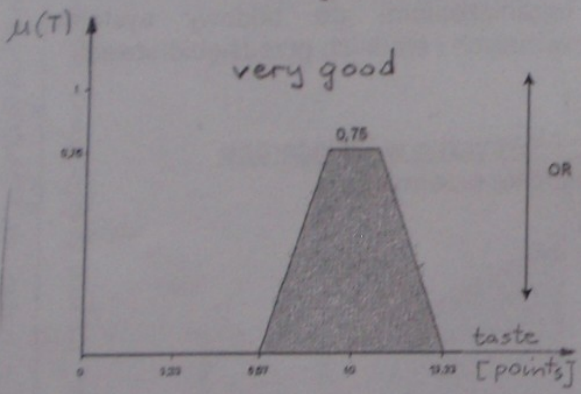
R1: IF(sugar am. small) THEN(taste medium)
S = 2.25 spoons



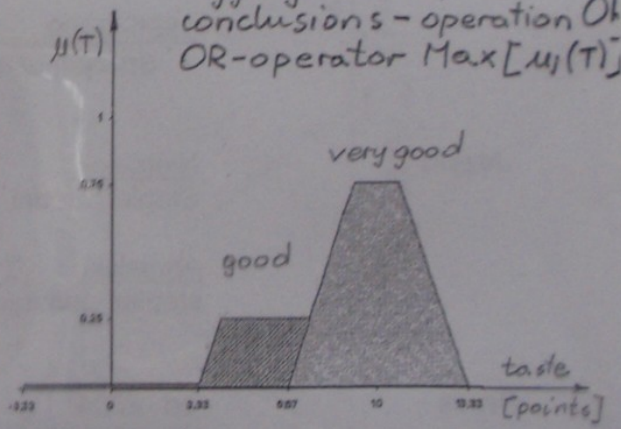
R4: IF(sugar amount very great)
THEN(taste poor)
S = 2.25



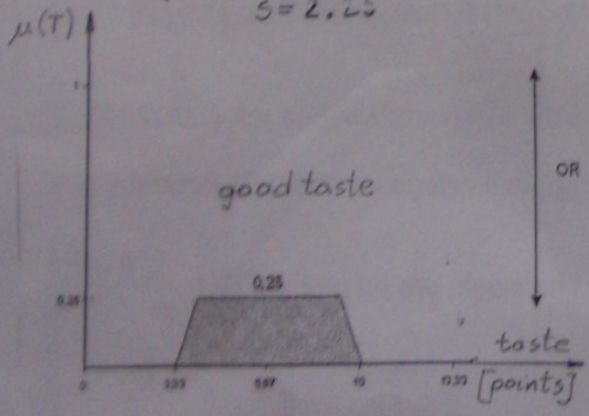
R2: IF(sugar amount average)
THEN(taste very good)



Aggregation of all conclusions - operation of OR-operator $\text{Max}[\mu_i(T)]$

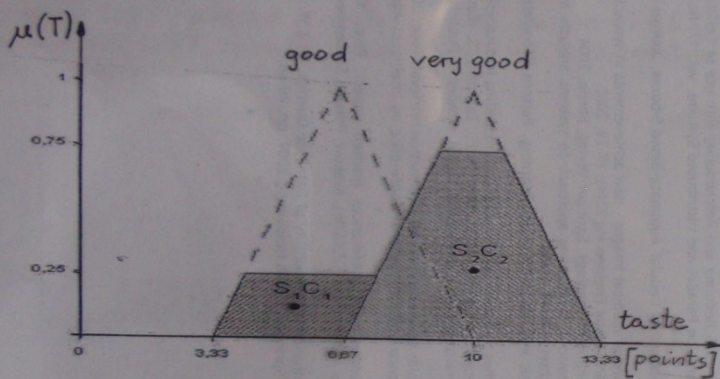


R3: IF(sugar amount great)
THEN(coffe taste good)
S = 2.25



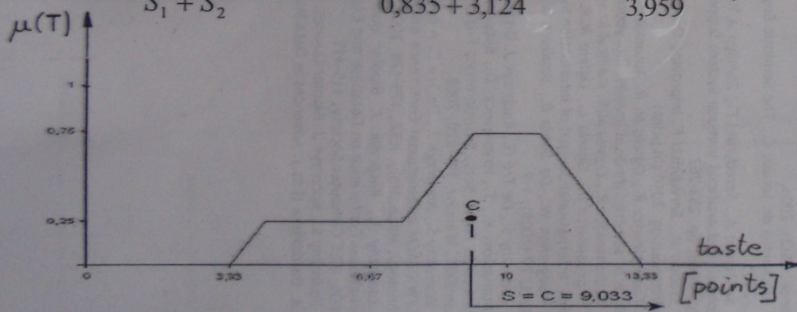
Defuzzification of the aggregated, general conclusion

Determining of the one-value representation with Center of Gravity Method



- S_i - area of the activated component i
- M_i - momentum of the component i
 $M_i = S_i \cdot C_i$
- C_i - position of Center of Gravity of the component i
- C - position of the global C of G of the aggregated conclusion

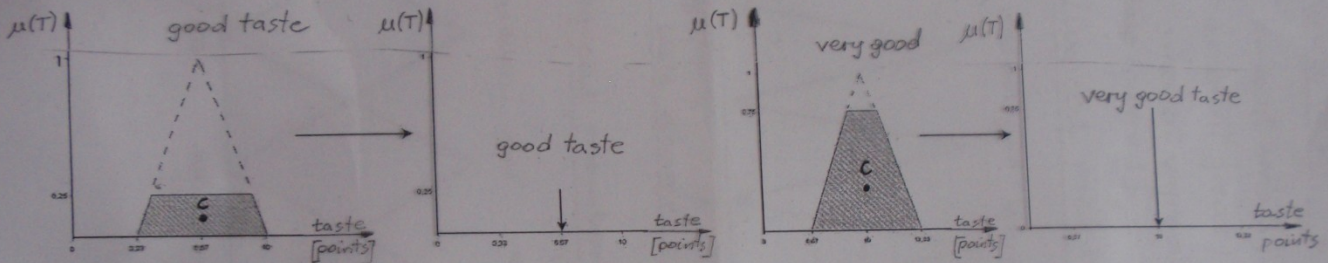
$$C = \frac{S_1 \cdot C_1 + S_2 \cdot C_2}{S_1 + S_2} = \frac{0,835 \cdot 5,416 + 3,124 \cdot 10}{0,835 + 3,124} = \frac{35,762}{3,959} = 9,033$$



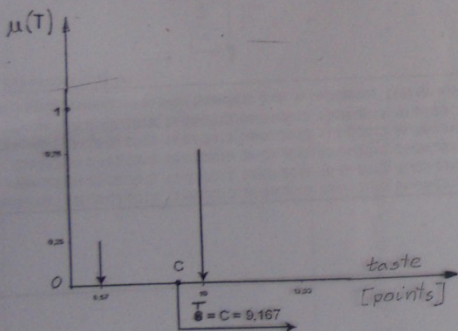
Defuzzification result:
 coffe taste $T = 9.033$ points
 corresponding to 2,25 sugar
 spoons

Defuzzification of the aggregated, general conclusion - with the Singleton-method

The membership function "good" taste, activated to 0.25, is replaced by a singleton also activated to degree 0.25 placed in its modal value $T=6.67$. The membership function "very good" taste activated to 0.75 is also replaced by a singleton activated to 0.75 and placed in its modal value $T=10$.



Defuzzification of the general conclusion



$$C = \frac{\mu_{\text{good}} \cdot C_{\text{good}} + \mu_{\text{v.good}} \cdot C_{\text{v.good}}}{\mu_{\text{good}} + \mu_{\text{v.good}}} = \frac{0.25 \cdot 6.67 + 0.75 \cdot 10}{0.25 + 0.75} = 9.167$$

Comment
the crisp representative C representing singleton conclusions is $T=C=9.167$ points.
Thus, singleton method gave very similar result as the Center of Gravity method which gave $T=9.033$

Typical errors made at construction of fuzzy models

Incomplete rule base - example

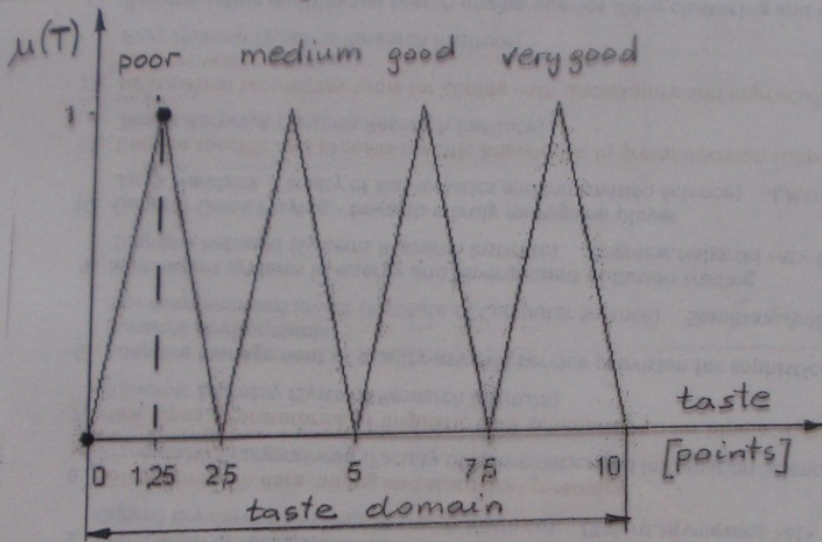
R1: IF(sugar amount small) THEN (taste medium)

R2: IF(sugar amount great) THEN (coffee taste good)

R3: IF(sugar amount v.great) THEN (taste poor)

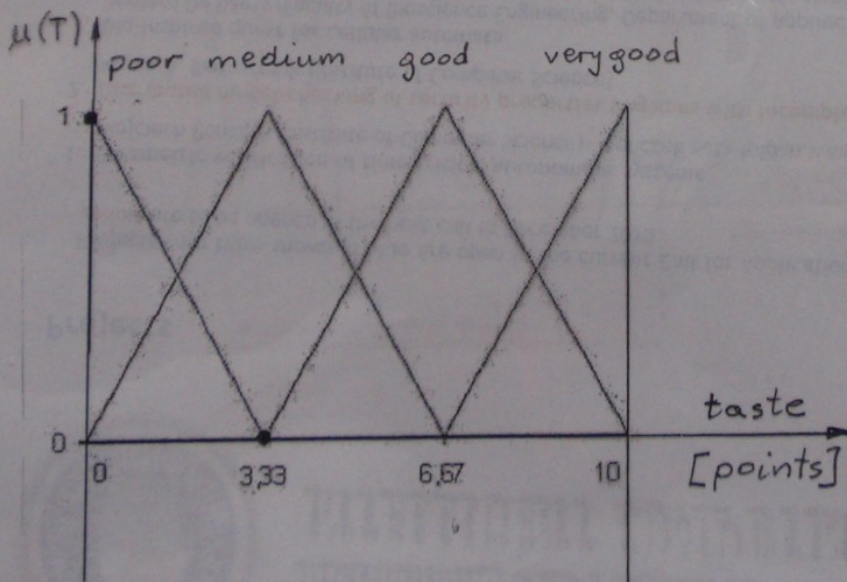
Why is this rule-base incorrect?

Incorrect, illogical membership function



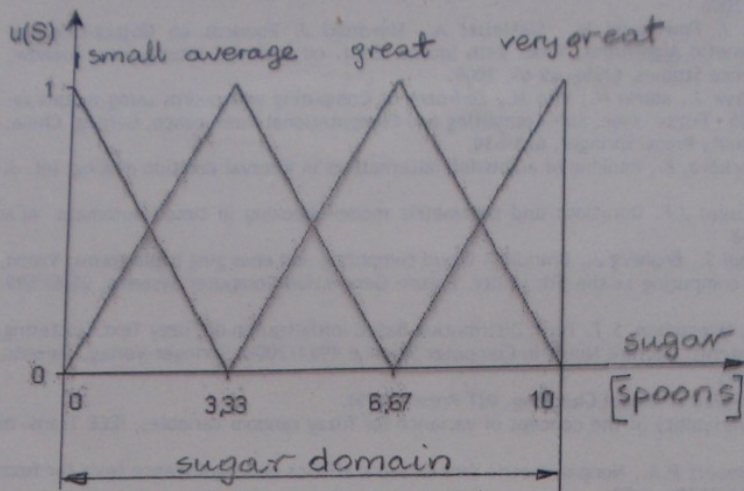
Why incorrect?

Correct, logical membership functions



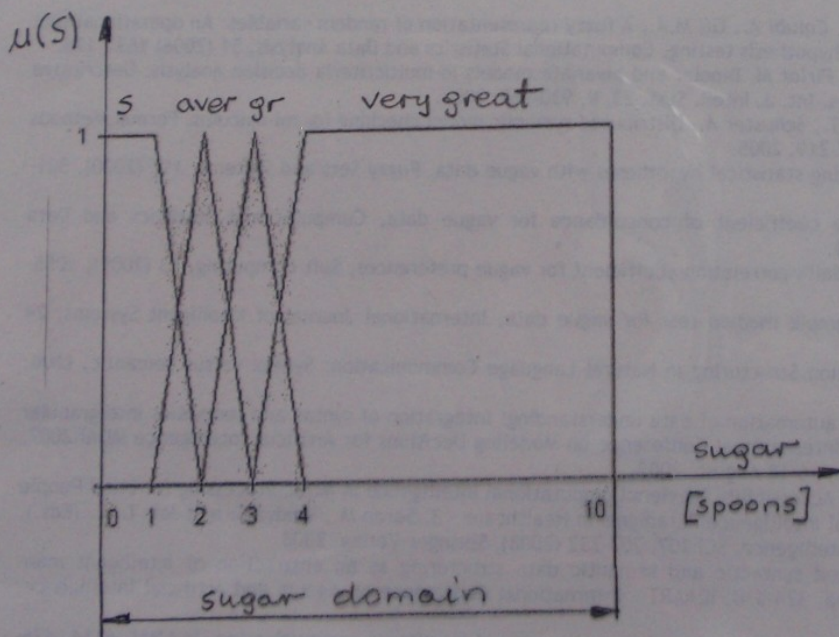
Why correct?

Incorrect membership functions - sugar



Why incorrect?

Correct membership functions of sugar



Why correct?