AHP RANK REVERSAL, NORMALIZATION AND AGGREGATION RULES*

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ABSTRACT

We analyze the Belton and Gear rank reversal problem within an axiomatic framework for deriving consistent weight ratios from pairwise ratio matrices and aggregating weights and ratio matrices. We show that rank reversal in the Analytic Hierarchy Process (AHP) is avoided when the output of the process is properly redefined as a weight-ratio matrix (rather than a normalized-weight vector) and multiplicative procedures — the geometric mean and the weighted-geometric-mean aggregation rule — which preserve the underlying mathematical structures are used.

Key Words : Aggregation Rules, AHP, Rank Reversal, Normalization, Decision Analysis.

RÉSUMÉ

Dans cet article on présent un cadre axiomatique qui permet d'obtenir des quotients consistents de poids dans la AHP (Analytic Hierarchy Process, Methode des Hierarchies Multiples) en partant d'une matrice de comparaisons par paires. Ensuite, on analyse la procédure d'agrégation. On montre qu'un renversement de l'ordre des poids finaux est évité par les opérations multiplicatives — la moyenne géométrique pour calculer les poids partiels des alternatives et l'agrégation géométrique pondérée pour obtenir les poids finaux. Ces opérations multiplicatives maintiennent la structure mathématique sous-jacente. L'example fameux de Belton et Gear illustre les résultats.

1. INTRODUCTION

We resolve Belton and Gear's rank reversal problem [4] in the Analytic Hierarchy Process (see [10]), by extending the axiomatic framework established in [1] to aggregation rules. The extended framework addresses the issues of extracting consistent weights from inconsistent weight matrices, normalization, rank reversal and inter-level aggregation rules in both the multiplicative and additive cases. Other issues which are not directly related to these topics (systems with feedback, appropriateness of the underlying scale, *etc.*) will be dealt with in future work. This is an *axiomatic* (*i.e. mathematical*) framework, which enables us to gain insight into problematic aspects of the underlying structure and to identify a consistent variant of the AHP. The framework does not address behavioral issues and is therefore independent of the way in which decision makers express their preferences. In other words, assuming that preferences have been expressed in the standard AHP format of estimated weight ratios (weight differences in the additive case), we provide a consistent axiomatic framework for extracting and aggregating weights from the data.

Because it appears from Belton and Gear's discussion that the reason for the rank reversal phenomenon is improper normalization of the weight vectors, earlier work (*e.g.* Belton and Gear [4], Harker and Vargas [8] and Saaty and Vargas [11], [12]) concentrated on

- proposing a normalization immune to rank reversal;
- proving that previously proposed normalizations are not immune to rank reversal;
- legitimizing rank reversals.

More recently, the exchange [14,6,13,9,7] in the March 1990 issue of *Management Science* is a clear indicator of the importance of resolving this controversy, which we do by showing that rank reversal can be avoided if a multiplicative aggregation rule is used and normalized weight vectors are

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replaced with weight-ratio matrices. (While the underlying presumption that one can extract absolute weights from weight ratios has never been challenged in the literature, it is clear that one can at most hope to retrieve consistent *weight ratios* — as opposed to *absolute* weights — from approximate *weight ratios.*) Naturally, the structure established in [1,3] provided us the clues needed to solve the rank reversal problem; the solution, in turn, confirms the correctness of this structure.

2. THE RANK REVERSAL PROBLEM

Belton and Gear consider in their Example 1 the three judgement matrices over three alternatives A, B and C:

$$\begin{pmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 9 \\ 1 & 1/9 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 1 \\ 1/9 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 8/9 & 8 \\ 9/8 & 1 & 9 \\ 1/8 & 1/9 & 1 \end{pmatrix},$$

and their corresponding normalized weight vectors

$$\begin{pmatrix} 1/11 \\ 9/11 \\ 1/11 \end{pmatrix}, \quad \begin{pmatrix} 9/11 \\ 1/11 \\ 1/11 \end{pmatrix}, \quad \begin{pmatrix} 8/18 \\ 9/18 \\ 1/18 \end{pmatrix}.$$

These vectors are combined (by taking their arithmetic mean) to produce the overall weight vector

$$w^* = \frac{1}{3}\begin{pmatrix} 1/11 \\ 9/11 \\ 1/11 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 9/11 \\ 1/11 \\ 1/11 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 8/18 \\ 9/18 \\ 1/18 \end{pmatrix} = \begin{pmatrix} 0.451 \\ 0.470 \\ 0.079 \end{pmatrix}. \tag{1}$$

In Example 2 they introduce an additional alternative, D, with judgement matrices

$$\begin{pmatrix} 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \\ 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 9 & 9 & 9 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 8/9 & 8 & 8/9 \\ 9/8 & 1 & 9 & 1 \\ 1/8 & 1/9 & 1 & 1/9 \\ 9/8 & 1 & 9 & 1 \end{pmatrix},$$

and corresponding normalized weight vectors

$$\begin{pmatrix} 1/20 \\ 9/20 \\ 1/20 \\ 9/20 \end{pmatrix}, \quad \begin{pmatrix} 9/12 \\ 1/12 \\ 1/12 \\ 1/12 \end{pmatrix}, \quad \begin{pmatrix} 8/27 \\ 9/27 \\ 1/27 \\ 9/27 \end{pmatrix},$$

yielding the overall weight vector

$$w = \frac{1}{3}\begin{pmatrix} 1/20 \\ 9/20 \\ 1/20 \\ 9/20 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 9/12 \\ 1/12 \\ 1/12 \\ 1/12 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 8/27 \\ 9/27 \\ 1/27 \\ 9/27 \end{pmatrix} = \begin{pmatrix} 0.365 \\ 0.289 \\ 0.057 \\ 0.289 \end{pmatrix}. \tag{2}$$

They then observe that $w_1^* < w_2^*$ but $w_1 > w_2$ so that the two sets of rankings are not consistent: the rank of A and B is reversed as a result of the inclusion of alternative D, even though the pairwise weight ratios associated with alternatives A, B, and C are unchanged and D is, in fact, a copy of B.

Clearly, if the AHP procedure generates rank reversals in the case of consistent input matrices, rank reversal can be expected in the inconsistent case as well. The following examples demonstrate this point. A straightforward calculation shows that the judgement matrices

$$\begin{pmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 8 \\ 1 & 1/8 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 8/9 \\ 1/9 & 9/8 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 8/9 & 8 \\ 9/8 & 1 & 8 \\ 1/8 & 1/8 & 1 \end{pmatrix},$$

yield the overall weight vector

$$v^* = \begin{pmatrix} 0.455 \\ 0.461 \\ 0.084 \end{pmatrix}. \tag{3}$$

When alternative D is added as above, the judgement matrices are

$$\begin{pmatrix} 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 8 & 1 \\ 1 & 1/8 & 1 & 1/8 \\ 9 & 1 & 8 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 9 & 9 & 9 \\ 1/9 & 1 & 8/9 & 1 \\ 1/9 & 9/8 & 1 & 9/8 \\ 1/9 & 1 & 8/9 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 8/9 & 8 & 8/9 \\ 9/8 & 1 & 8 & 1 \\ 1/8 & 1/8 & 1 & 1/8 \\ 9/8 & 1 & 8 & 1 \end{pmatrix},$$

and the overall weight vector is

$$v = \begin{pmatrix} 0.367 \\ 0.286 \\ 0.061 \\ 0.286 \end{pmatrix}.$$

Again, $v_1^* < v_2^*$ but $v_1 > v_2$ and the rank of A and B is reversed as a result of the inclusion of alternative D.

3. NO NORMALIZATION CAN PREVENT RANK REVERSAL

For the example above, we see by comparing equations (1) and (2) that the difference in relative magnitude of the components of the overall weight vectors w^* and w is due to the normalization factors applied to the weight vectors. Explicitly, if the vector z is obtained from the (positive) vector x by the additive normalization $z_j = x_j / \sum_{i=1}^n x_i$, then any component z_j is sensitive to changes in any other component x_k. Clearly, the same is true when a component is added to x — the essence of normalizing a vector is to adjust some (or all) of its components on the basis of the magnitude of *other* components. Since this is true for *all* normalizations, it follows that there does not exist any normalization which avoids rank reversal:

Theorem 1.

For any normalization, there exists a set of vectors exhibiting rank reversal.

Proof.

Consider the vectors

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

and

$$\begin{pmatrix} x_1/m(x) \\ x_2/m(x) \\ \vdots \\ x_n/m(x) \end{pmatrix} + \begin{pmatrix} y_1/m(y) \\ y_2/m(y) \\ \vdots \\ y_n/m(y) \end{pmatrix},$$

where $m(x)$ is a fixed but otherwise arbitrary normalization applied to the vector x. (Typical examples are: $m(x) = \sum_{i=1}^n x_i$, $m(x) = \prod_{i=1}^n x_i$ and $m(x) = \max_i \{x_i\}$.) Clearly, for any arbitrary choice of $m(x)$, by adjusting the values of x_n and y_n, one can always select vectors x, y for which $x_1 + y_1 > x_2 + y_2$ while $x_1/m(x) + y_1/m(y) < x_2/m(x) + y_2/m(y)$. •

In particular, for the normalization proposed by Belton and Gear in [4], the components of the normalized vector are unchanged until a certain threshold is attained. But this normalization too, as pointed out by Saaty and Vargas in [12], is subject to rank reversal.

4. NOTATION

The matrices below are $n \times n$, vectors are n-dimensional, and:

$A = (a_{ij})$ is a *pairwise multiplicative matrix* if $0 < a_{ij} = 1/a_{ji}$;
$w = (w_k)$ is a *multiplicative weight vector* if $w_k > 0$ and $\prod_{k=1}^{n} w_k = 1$;
$C = (c_{ij})$ is a *multiplicative consistent matrix* if $c_{ij} = w_i/w_j$ for some multiplicative weight vector w;
$A^\times, w^\times$ and $C^\times$ are the sets of all pairwise multiplicative matrices, multiplicative weight vectors and multiplicative consistent matrices, respectively;
$f^\times$ is the set of all mappings from $A^\times$ to $w^\times$.
$\phi^\times$ is the set of all mappings from $A^\times$ to $C^\times$.

Note that $A^\times$, $w^\times$ and $C^\times$ are groups under componentwise multiplication and $C^\times$ is isomorphic to $w^\times$.

5. REPRESENTATION IN $C^\times$

The weight vectors retrieved from pairwise comparison matrices in the AHP are determined only up to a multiplicative factor. These vectors can therefore be represented by normalized proxies or, since $C^\times$ is isomorphic to $w^\times$, by matrices of the form (w_i/w_j). For the purpose of studying the problem of retrieving weights from pairwise comparison matrices, the choice of representation is immaterial (see *e.g.* [1] or [3]). However, it follows from the above that these representations are *not equivalent* as far as aggregating weights and ratio matrices is concerned. More importantly, weight ratios are preserved when normalized weight vectors are replaced by weight ratio matrices because no extraneous normalization factors are introduced. Indeed, the matrix representation for the examples above yields

$$W^* = \frac{1}{3}\begin{pmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 9 \\ 1 & 1/9 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 1 \\ 1/9 & 1 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & 8/9 & 8 \\ 9/8 & 1 & 9 \\ 1/8 & 1/9 & 1 \end{pmatrix} \tag{1'}$$

$$= \begin{pmatrix} 1 & 3.333 & 6 \\ 3.412 & 1 & 6.333 \\ 0.412 & 0.407 & 1 \end{pmatrix}$$

and

$$W = \frac{1}{3}\begin{pmatrix} 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \\ 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & 9 & 9 & 9 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & 8/9 & 8 & 8/9 \\ 9/8 & 1 & 9 & 1 \\ 1/8 & 1/9 & 1 & 1/9 \\ 9/8 & 1 & 9 & 1 \end{pmatrix} \tag{2'}$$

$$= \begin{pmatrix} 1 & 3.333 & 6 & 3.333 \\ 3.412 & 1 & 6.333 & 1 \\ 0.412 & 0.407 & 1 & 0.407 \\ 3.412 & 1 & 6.333 & 1 \end{pmatrix}.$$

The relative ranking of alternatives A and B is unchanged since the numerical value of w_1/w_2 is preserved — in fact, W^* is a principal minor of W (that is, W^* is obtained by deleting certain rows and the same numbered columns of W).

6. AXIOMS FOR AGGREGATION RULES

Let F be an aggregation rule for combining l positive weights and $n \times n$ matrices. This rule is then a mapping from the set of all $\{\lambda_1, \ldots, \lambda_l; A_1, \ldots, A_l\}$ to the set of positive $n \times n$ matrices.

Guided by the examples above and the underlying multiplicative structures, we postulate axioms for aggregation rules and prove that the weighted-geometric-mean aggregation rule satisfies these axioms and, consequently, is immune to rank reversal.

Axiom 1.

The aggregation rule F satisfies

$$F(\lambda_1,\ldots,\lambda_l; P(A_1),\ldots,P(A_l)) = P(F(\lambda_1,\ldots,\lambda_l; A_1,\ldots,A_l)),$$

where the operator P denotes taking a (fixed) principal minor of the appropriate matrices.

Axiom 2.

If the input matrices of F are consistent, so is its output matrix:

$$A_k \in C^{\times} \quad k = 1,\ldots,l \quad \Rightarrow \quad F(\lambda_1,\ldots,\lambda_l; A_1,\ldots,A_l) \in C^{\times}.$$

Axiom 3.

For some $\phi \in \phi^{\times}$,

$$F(\lambda_1,\ldots,\lambda_l; \phi(A_1),\ldots,\phi(A_l)) = \phi(F(\lambda_1,\ldots,\lambda_l; A_1,\ldots,A_l)).$$

The significance of Axiom 1 is that if $P(A)$ is a principal minor of A, then $P(A)$ and A represent the same judgment ratios over the subset of objects they have in common. It follows directly from the definition that if F satisfies Axiom 1, the weight ratios of the common objects are identical and therefore not affected by the inclusion of additional objects. Formally:

Theorem 2.

An aggregation rule satisfying Axiom 1 is not subject to rank reversal.

Axiom 2 is needed since an arbitrary aggregation rule may produce matrices which are neither consistent nor even pairwise multiplicative, as is the case for W^* and W in (1′) and (2′) above. That the arithmetic mean destroys this property is not surprising in view of our earlier analysis of the underlying algebraic structure (see Barzilai *et al.* [1,2,3]).

Axiom 3 means that if the input pairwise multiplicative matrices are converted — using the mapping ϕ — to their consistent representative matrices and the resulting multiplicative consistent matrices are aggregated using the rule F, the result obtained is the same as when the input pairwise multiplicative matrices are first aggregated using the rule F and the pairwise multiplicative matrix obtained in this manner is then converted to its consistent representative matrix using the mapping ϕ. In other words, the final result is independent of the order of operation.

Keeping in mind the underlying multiplicative structures, it is easy to extend the observations in §5:

Theorem 3.

*The weighted-geometric-mean aggregation rule**

$$F(\lambda_1,\ldots,\lambda_l; A_1,\ldots,A_l) = \prod_{k=1}^{l} A_k^{\lambda_k}$$

satisfies Axioms 1–3.

* *All operations are carried out componentwise.*

Proof.

For objects i and j belonging to the principal minor corresponding to the operator P, both sides of the equation defining Axiom 1 are given by

$$\prod_{k=1}^{l}(a_{ij})_k^{\lambda_k},$$

where $(a_{ij})_k$ denotes the ij element of A_k. Therefore, F satisfies Axiom 1.

Next, note that for $m = 1,\ldots,l$, $A_m \in C^{\times}$ is equivalent to $(a_{ij})_m(a_{jk})_m(a_{ki})_m = 1$. This implies

$$\prod_{m=1}^{l}(a_{ij})_m^{\lambda_m}\prod_{m=1}^{l}(a_{jk})_m^{\lambda_m}\prod_{m=1}^{l}(a_{ki})_m^{\lambda_m} = 1,$$

so that $F(\lambda_1,\ldots,\lambda_l; A_1,\ldots,A_l)$ is consistent. Hence F satisfies Axiom 2.

Finally, in conjunction with the geometric mean mapping defined by

$$f(A) = W = (w_{ij}) \qquad w_{ij} = \left(\frac{\prod_{k=1}^{n} a_{ik}}{\prod_{k=1}^{n} a_{jk}}\right)^{1/n},$$

the weighted-geometric-mean aggregation rule satisfies Axiom 3 since

$$\prod_{m=1}^{l}\left(\frac{\prod_{k=1}^{n}(a_{ik})_m^{1/n}}{\prod_{k=1}^{n}(a_{jk})_m^{1/n}}\right)^{\lambda_m} = \prod_{k=1}^{n}\left(\frac{\prod_{m=1}^{l}(a_{ik})_m^{\lambda_m}}{\prod_{m=1}^{l}(a_{jk})_m^{\lambda_m}}\right)^{1/n}.$$

which completes the proof •

To illustrate the above, the weighted-geometric-mean produces for the Belton and Gear examples the matrices

$$U^* = \begin{pmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 9 \\ 1 & 1/9 & 1 \end{pmatrix}^{1/3} \times \begin{pmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 1 \\ 1/9 & 1 & 1 \end{pmatrix}^{1/3} \times \begin{pmatrix} 1 & 8/9 & 8 \\ 9/8 & 1 & 9 \\ 1/8 & 1/9 & 1 \end{pmatrix}^{1/3} \qquad (1'')$$

$$= \begin{pmatrix} 1 & 0.961 & 4.160 \\ 1.040 & 1 & 4.327 \\ 0.240 & 0.231 & 1 \end{pmatrix}$$

and

$$U = \begin{pmatrix} 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \\ 1 & 1/9 & 1 & 1/9 \\ 9 & 1 & 9 & 1 \end{pmatrix}^{1/3} \times \begin{pmatrix} 1 & 9 & 9 & 9 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \\ 1/9 & 1 & 1 & 1 \end{pmatrix}^{1/3} \times \begin{pmatrix} 1 & 8/9 & 8 & 8/9 \\ 9/8 & 1 & 9 & 1 \\ 1/8 & 1/9 & 1 & 1/9 \\ 9/8 & 1 & 9 & 1 \end{pmatrix}^{1/3} \qquad (2'')$$

$$= \begin{pmatrix} 1 & 0.961 & 4.160 & 0.961 \\ 1.040 & 1 & 4.327 & 1 \\ 0.240 & 0.231 & 1 & 0.231 \\ 1.040 & 1 & 4.327 & 1 \end{pmatrix};$$

U^* and U are (consistent) pairwise multiplicative and U^* is a principal minor of U. Thus, the ranking of alternatives A, B and C under U^* and U is identical.

7. CONCLUSIONS

By adding new axioms, the axiomatic framework for deriving consistent weight ratios from pairwise ratio matrices developed in [1] has been extended to deal with aggregation rules for combining weights and ratio matrices. Within this extended framework, we have demonstrated that the AHP rank reversal controversy has centred on the wrong issues. Rank reversal is neither a fatal flaw of the AHP, nor a desirable property of it — when the correct structure is used, rank reversal does not occur. It is a symptom of inherent problems with the AHP: the output of the process is improperly defined as weights rather than weight ratios, and non-multiplicative procedures (the weighted arithmetic mean and the eigenvector) are imposed on an intrinsically multiplicative structure.

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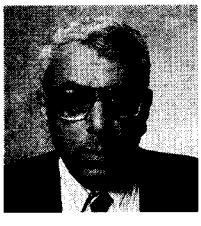

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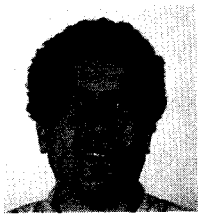

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