Evolution Strategies (1+1)-ES for test functions

1 Evolution Strategies in version (1+1)-ES

$parent \leftarrow initialize(domain, function)$
$g \leftarrow 0$
while $g < max\_num\_gen$ **do**
  $s \leftarrow s\_mutation\,(s)$
  $offspring \leftarrow y\_mutation\,(parent, s)$
  $fitness\_function\,(offspring)$
  $parent \leftarrow selection(offspring, parent)$
  $g \leftarrow g + 1$
end while
return $parent$

2 Algorithm description

- $initialize(domain, function)$ returns an individual (parent) which could be structure/object:
  - x — vector with d numbers which represents optimization function arguments - point in the domain space. $\Re^n$). Initialized randomly from uniform distribution and the range = $domain$.
  - s — vector with d numbers which represents standard deviation to control the mutation strength in every dimention. Initialized randomly from uniform distribution and the range = $[0.1 : 10]$.
  - function — fitness function (calculated from the formula e.g. Schwefel function)

  $domain$ is a given domain for a specific test function (benchmark) that will be solved by ES.

- $s\_mutation\,(s)$
  1. perform the (1+1)-ES for a number G generations (e.g $G = 5$)
     (a) keep s constant during that period
     (b) count the number G_s of successful mutation (when $fitness\_function(child) < fitness\_function(parent)$ during that period

2. calculate $P_s = \frac{G_s}{G}$
3. change s (every s_i):

$$s_i := \begin{cases} s_i/a, & \text{if } P_s > 1/5 \\ s_i \cdot a, & \text{if } P_s < 1/5 \\ s_i, & \text{if } P_s = 1/5 \end{cases}$$

4. goto 1

Parameter $0 \leq a \leq 1$, a recommended value is $a = 0.85$.

- $y\_mutation(parent, s)$

  for $i = 1$ **to** $length\,(parent)$ **do**
  $\quad mutant_i \leftarrow y_i + s_i \cdot \aleph\,(0,1)$
  end for
  return $mutant$

  where $\aleph\,(0,1)$ is the randomly generated value from normal distribution.

- max_num_gen - maxim number of generations e.g. 10000

3 Benchmarks

To test the algorithm the following function should be used:

1. RASTRIGIN FUNCTION
   - `http://www.sfu.ca/~ssurjano/rastr.html`,
   - $domain = [-5.12, 5.12]$
   - Global minimum $f(0, 0, \ldots, 0) = 0$
2. GRIEWANK FUNCTION
   - `http://www.sfu.ca/~ssurjano/griewank.html`,
   - $domain = [-600, 600]$
   - Global minimum $f(0, 0, \ldots, 0) = 0$
3. SPHERE FUNCTION
   - `http://www.sfu.ca/~ssurjano/spheref.html`,
   - $domain = [-5.12, 5.12]$
   - Global minimum $f(0, 0, \ldots, 0) = 0$
4. ZAKHAROV FUNCTION
   - `http://www.sfu.ca/~ssurjano/zakharov.html`,
   - $domain = [-5, 10]$

- Global minimum $f(0, 0, \ldots, 0) = 0$

5. EASOM FUNCTION

   - `http://www.sfu.ca/~ssurjano/easom.html`,
   - $domain = [-100, 100]$
   - Global minimum $f(\pi, \pi) = -1$

6. STYBLINSKI-TANG FUNCTION

   - `http://www.sfu.ca/~ssurjano/stybtang.html`,
   - $domain = [-5, 5]$
   - Global minimum $f(-2.903534, \ldots, -2.903534) = -39.16599d$, where d is dimention